PERTH MODERN SCHOOL

Year 11 Mathematics Specialist – 2020

INVESTIGATION 2 – Take-home section

Mathematical Induction and Trigonometry

PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.

FULL NAME ____ Date Received ______________________

Syllabus

2.3.4 develop the nature of inductive proof, including the 'initial statement' and inductive step

2.3.5 prove results for sums, such as $1+4+9...+n^2=\frac{n(n+1)(2n+1)}{6}$ for any positive integer n

2.1.3 prove and apply the angle sum, difference, and double angle identities

2.1.5 prove and apply the Pythagorean identities

2.1.6 prove and apply the identities for products of sines and cosines expressed as sums and differences

Question 1

Prove, by Mathematical Induction, that for all positive integers *n*,

$$\sin(x)+\sin(3x)+\sin(5x)+\cdots+sin[(2n-1)x]=\frac{1-cos\,(2nx)}{2sin\,(x)}$$

where $sin\,(x)\neq 0.$

Question 2

Prove, by Mathematical Induction, that for all positive integers *n*,

$$2\,cos(2x)+2\,cos(4x)+2\,cos(6x)+\cdots+2\,cos(2nx)=\frac{sin[(2n+1)x]}{sin(x)}-1$$

where $sin\,(x)\neq 0.$

Question 3

For all positive integers of *n*, use Mathematical Induction to prove that

$$\cos(2x)\times cos(2^2x)\times\ldots\times cos(2^nx)=\frac{sin\,(2^{n+1}x)}{2^n sin\,(2x)}$$

where $sin\,(x)\neq 0.$

Validation Test Conditions:

- Calculator Assumed and a formula sheet will be provided.
- Two double-sided A4 sheets of notes allowed for the combined Investigation 2 Validation / Test 2 assessment which will be held on Monday of week 4 (10 August)